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On dark stars, galactic rotation curves and fast radio bursts

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Content

Short introduction to General Relativity

Models of compact massive objects

- dark stars
- wormholes
- white holes
- Planck stars

RDM-stars

Comparison of RDM-model with experiment

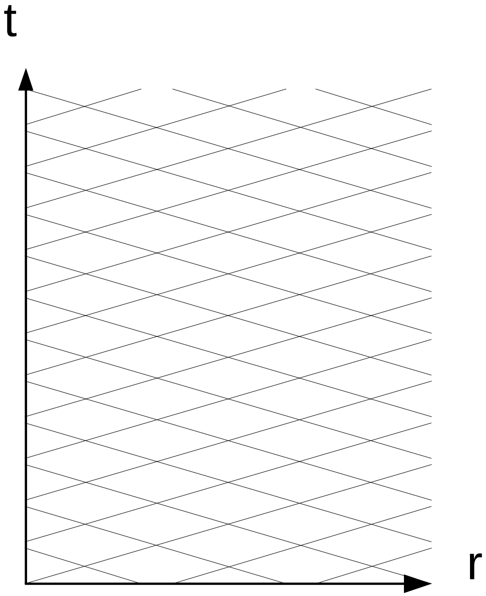
- fast radio bursts
- galactic rotation curves

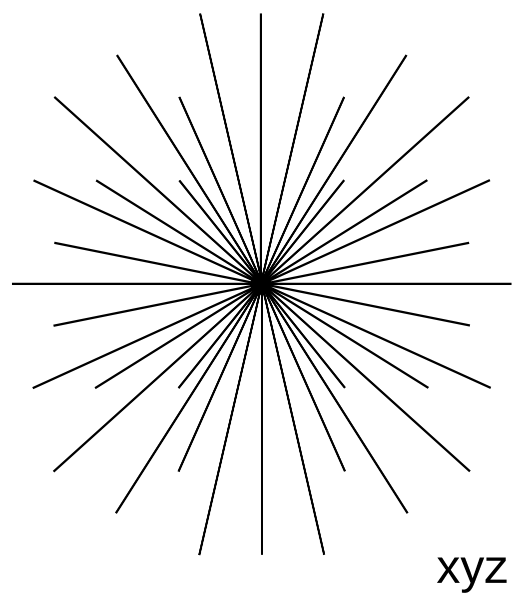
- arXiv:1701.01569
- arXiv:1812.11801
- arXiv:1903.09972

Dark Stars

- also known as ***quasi black holes***, boson stars, gravastars, fuzzballs ...
- solutions of general theory of relativity, which first follow Schwarzschild profile and then *are modified*
- outside are similar to black holes, inside are constructed differently (depending on the model of matter used)
- review of the models: Visser et al., ***Small, dark, and heavy: But is it a black hole?***, arXiv: 0902.0346
- our contribution to this family: *RDM stars* (quasi black holes coupled to Radial Dark Matter)

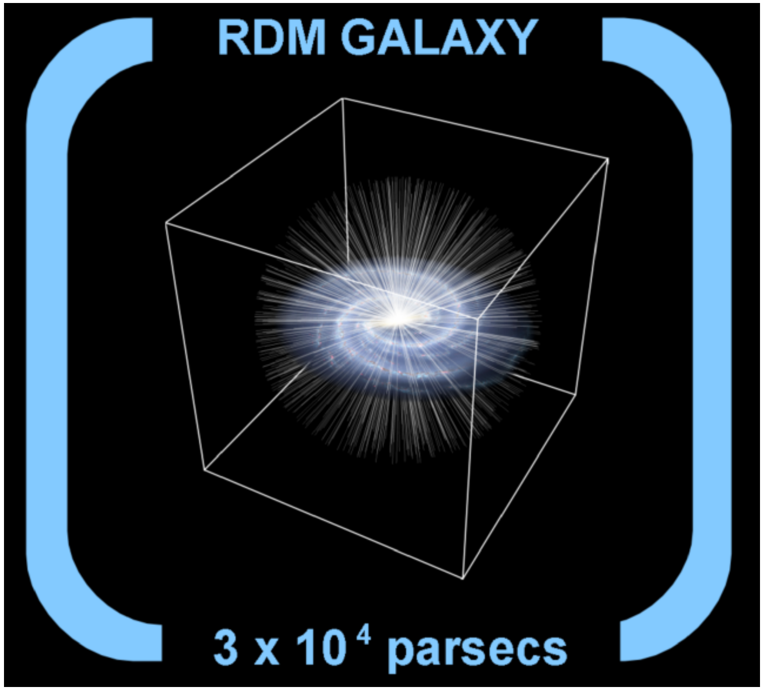
Stationary solution, including T-symmetric superspositon of ingoing and outgoing radially directed flows of dark matter





Galactic model with radial dark matter

- The simplest model of a spiral galaxy
- 30kpc level (for MW)
- Dark matter flows radially converge towards the center of the galaxy
- The limit of weak gravitational fields, one-line calculation: $\rho \sim r^{-2}$, $M \sim r$, $v^2 = GM / r = Const$
- Qualitatively correct behavior of galactic rotation curves (asymptotically flat shape at large distances)
- The orbital velocity of the stars and interstellar gas consisting of Kepler and constant terms
- Q1: What is happening in the center of the galaxy? (requires the calculation in the limit of strong fields)



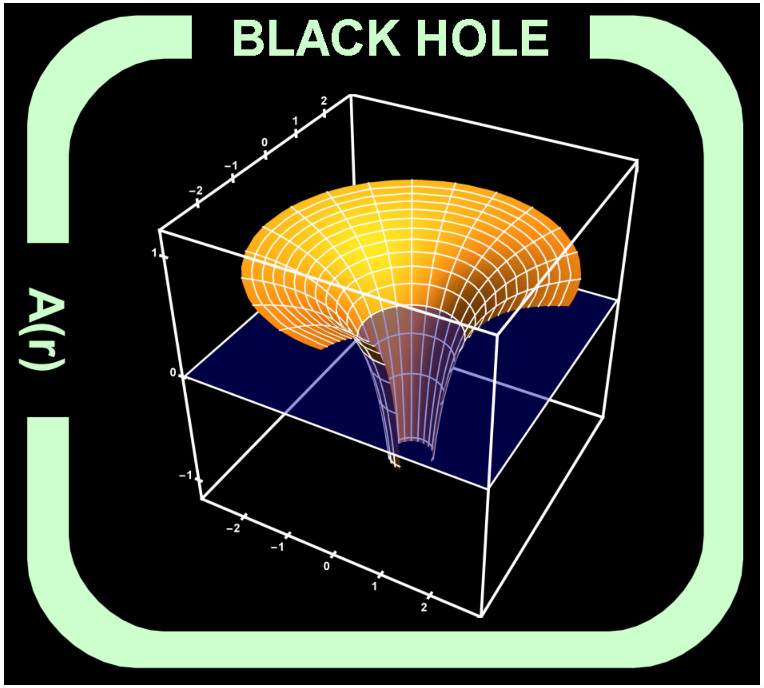
$$v^2 = GM/r + Const$$

- Q2: How to describe the deviation of rotation curves from the flat shape? (the model of distributed RDM-stars will be considered)

"Ordinary" black hole

- Schwarzschild solution
- spherical coordinate system
- $r = r_0$ - Event Horizon
- after crossing the horizon A,B reverse sign,

  r,t roles are interchanged
- radial movement toward the center becomes equivalent to the increasing time
- => material objects fall onto central singularity
- above the horizon A-profile controls slowdown of time and the wavelength shift (0<A<1 red, A>1 blue); B - the deformation of the radial coordinate, D - deformation angular coordinates



$$A = 1 - r_0/r, \;\; B = A^{-1}, \;\; D = 1$$

$$ds^2 = -Adt^2 + Bdr^2 + Dr^2(d\theta^2 + \sin^2\theta \; d\phi^2)$$

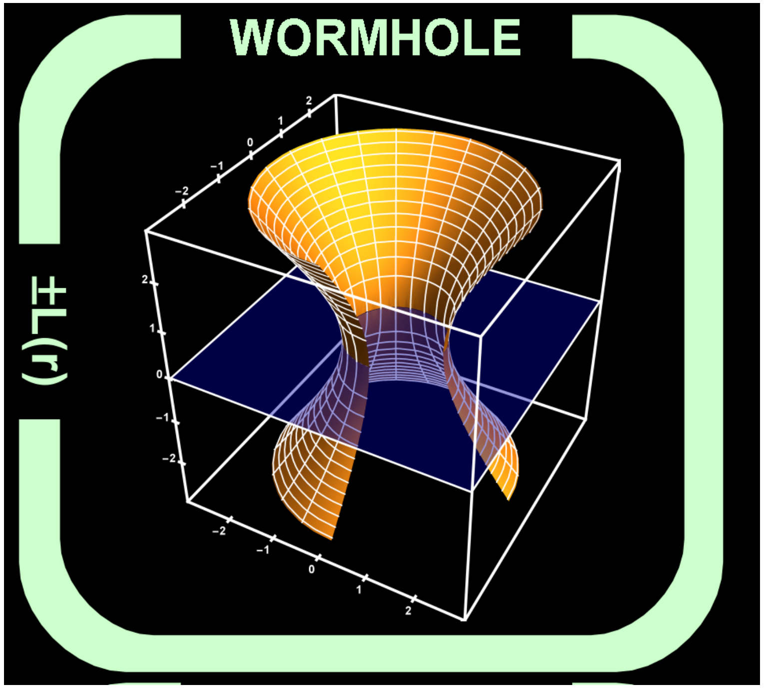
metric, the square of the distance between points in the curved space-time

M.Blau, Lecture Notes on General Relativity, University of Bern 2018

The Wormole

- type MT (Morris-Thorne model)
- no event horizon
- a tunnel connecting 2 universes or 2 sites of a single universe
- requires exotic matter ($\rho+p<0$)
- $B \rightarrow \infty$, $A>0$ is finite, L is finite

$$L(r) = \int dr \sqrt{B(r)}$$

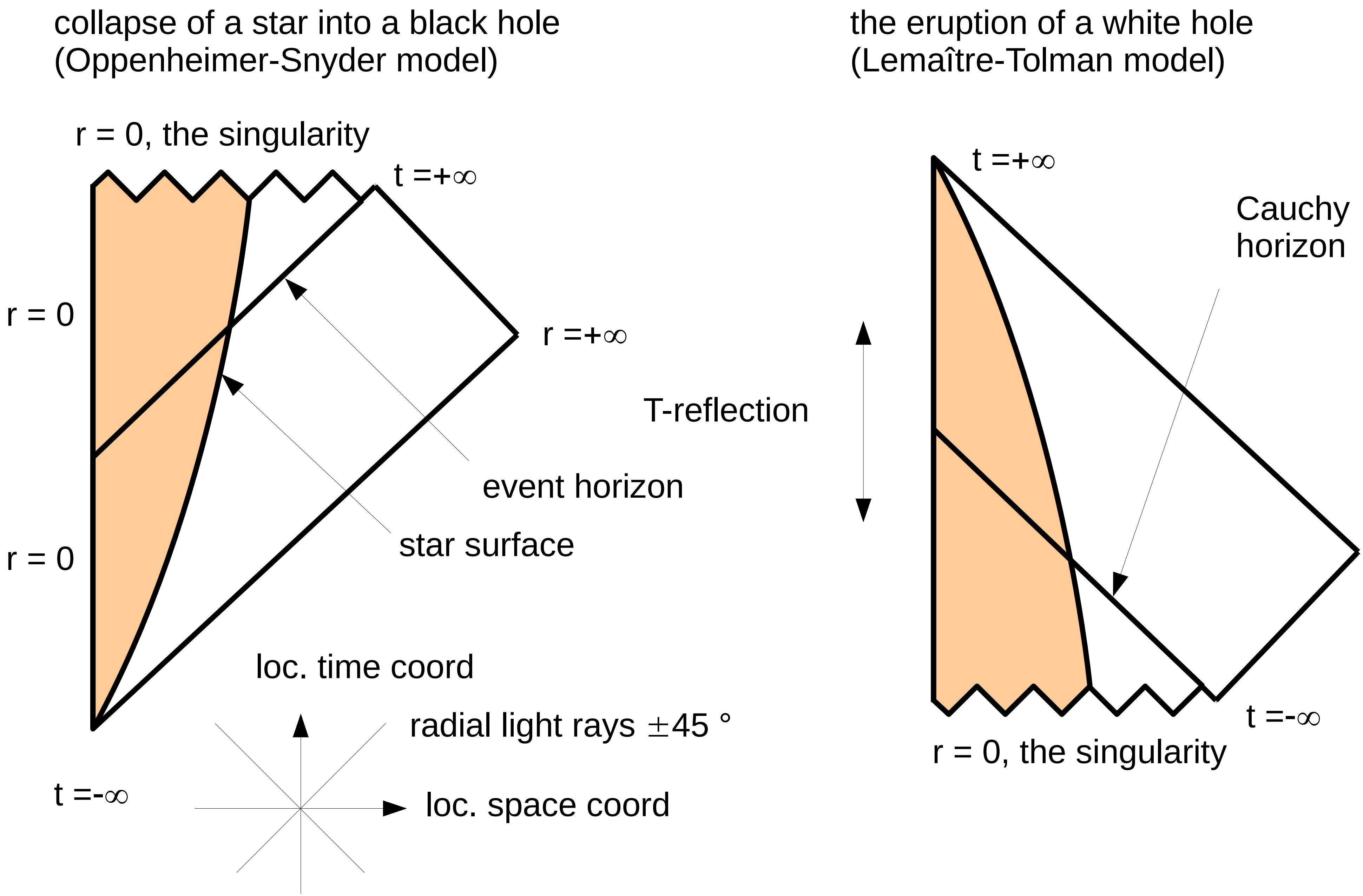


- a specific example:

$$A = 1 - r_0/r + \alpha/r^2,\ \ B = (1 - r_0/r)^{-1},\ \ D = 1$$

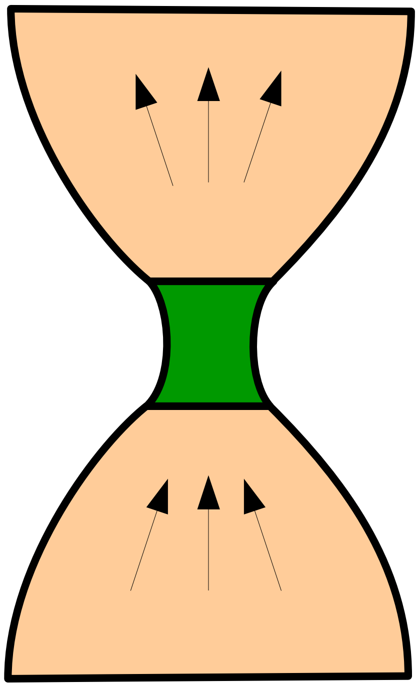
M.Visser, Lorentzian Wormholes: from Einstein to Hawking, Springer 1996

White hole on Penrose diagram



Planck stars

- Planck density: $\rho_P = c^5/(\hbar G^2) =$ **$5x10^{96}$kg / m^3**
- straightforward estimation for the Planck density core of R = 10km radius, the mass $M = (4/3)\pi R^3 \rho_P = 2x10^{109}$kg, gravitational radius: $Rs = 2GM/c^2 = 3x10^{82}$m, compare to the mass and radius of the observable universe Muni = 10^{53}kg, Runi = $4x10^{26}$m (such a star will immediately cover the universe by its gravitational radius, with a large margin)
- however, quantum gravity (QG) gives a correction to the density: $\rho_X = \rho\,(1-\rho/\rho_P)$
- $\rho = \rho_P \Rightarrow \rho_X = 0$ at Planck density the gravity is switched off
- $\rho > \rho_P \Rightarrow \rho_X < 0$ in excess of Planck density the effective negative mass appears (**exomatter**), gravitational repulsion (antigravity)
- the models: Rovelli-Vidotto (2014), Barceló et al. (2015)
- for an external observer strong grav. time dilation is applied
- estimation of the re-collapse time depends on the size, t~13.8 bln.years for microholes of r ~ $2x10^{-4}$m
- (one of the possible mechanisms of *fast radio bursts*)



QG bounce:
collapse replaced
by extension,
black hole turns white

General theory of relativity, a brief introduction

- spacetime - 4D manifold
- x^μ - arbitrary coordinates, e.g., linear Minkowski, or curved spherical, cylindrical, etc.
- $g_{\mu\nu}(x)$ - the metric tensor

  coordinate-dependent symmetric 4x4 matrix

  eigenvalues of signature (+++ -) 3 space coords + 1 time
- $g^{\mu\nu}(x)$ - inverse matrix
- squared distance between points in a general form: $$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$
- summation over repeated indices everywhere assumed
- indices: subscript - covariant, superscript - contravariant
- raising / lowering index operations,

  tensor transformation rules under change of coordinates:

$$u_\mu = g_{\mu\nu} u^\nu,\ u^\mu = g^{\mu\nu} u_\nu,\ G_{\mu\nu} = G^{\alpha\beta} g_{\alpha\mu} g_{\beta\nu}$$

$$g_{\alpha\beta}(y) = g_{\mu\nu}(x) J^\mu_\alpha J^\nu_\beta \qquad J^\mu_\alpha = \partial x^\mu / \partial y^\alpha$$

Jacobi matrix

General theory of relativity, a brief introduction

covariant derivative

coordinate derivative

Christoffel symbols

$$\nabla_\mu u^\lambda = \partial_\mu u^\lambda + \Gamma^\lambda{}_{\mu\nu} u^\nu, \;\; \partial_\alpha = \partial/\partial x^\alpha,$$

$$\Gamma^\lambda{}_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}),$$

Riemann tensor

$$R^\lambda{}_{\mu\nu\sigma} = (\partial_\nu \Gamma^\lambda{}_{\mu\sigma} + \Gamma^\eta{}_{\mu\sigma}\Gamma^\lambda{}_{\eta\nu}) - (\sigma \leftrightarrow \nu),$$

$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}, \;\; R = g^{\mu\nu} R_{\mu\nu},$$

Ricci tensor, Ricci scalar

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R.$$

Einstein tensor

General theory of relativity, a brief introduction

Einstein eqs: relate gravitational field to the distribution of matter

$$G^{\mu\nu} = 8\pi G/c^4 \cdot T^{\mu\nu},\ \ u^\nu \nabla_\nu u^\mu = 0,\ \ \nabla_\mu \rho u^\mu = 0$$

=> self-consistent PDE system

geodesic eqs: relate matter distr. to the gravitational field

$$4\pi G = 1, c = 1$$

Geometric System of Units
(sometimes G = c = 1 are chosen)

matter distribution in the RDM model: the energy-momentum tensor

$$T^{\mu\nu} = \rho(u_+^\mu u_+^\nu + u_-^\mu u_-^\nu),\ \ u_\pm = (\pm u^t, u^r, 0, 0)$$

mass density > 0
pressure p = 0

radial velocity flows: incoming / outgoing

Note: steady-state solution requires energy balance of the flows

- zeroing total energy flux through r-spheres: $T^{tr} = 0$
- satisfied in the particular case with T-symmetric flow, above
- (Necessary to investigate): general case

Derivation of RDM equations, computer algebra

- complex calculations, for example, Riemann tensor 4^4= 256 components
- substitution, differentiation, algebraic simplifications
- convenient to use a system of analytical computations
- example calculation in Mathematica

Algorithm Einstein(n,x,g):

```
ginv = Simplify[Inverse[g]];          (* Inv.metr.tensor *)
gam = Simplify[ Table[
       (1/2)*Sum[ ginv[[i,s]]*(D[g[[s,j]],x[[k]]]
         +D[g[[s,k]],x[[j]]]-D[g[[j,k]],x[[s]]]),
        {s,1,n} ],
      {i,1,n},{j,1,n},{k,1,n} ] ];     (* Christoff. symb. *)
R4 = Simplify[ Table[
     D[gam[[i,j,l]],x[[k]]]-D[gam[[i,j,k]],x[[l]]]
     + Sum[ gam[[s,j,l]] gam[[i,k,s]]
       - gam[[s,j,k]] gam[[i,l,s]],
       {s,1,n} ],                      (* Riemann tensor *)
     {i,1,n},{j,1,n},{k,1,n},{l,1,n} ] ];
R2 = Simplify[ Table[
       Sum[ R4[[i,j,i,l]],{i,1,n} ],    (* Ricci tensor *)
     {j,1,n},{l,1,n} ] ];
R0 = Simplify[ Sum[
       ginv[[i,j]] R2[[i,j]],           (* Ricci scalar *)
     {i,1,n},{j,1,n} ] ];
G2 = Simplify[ R2 - (1/2) R0 g ]      (* Einstein tensor *)
```

J.B.Hartle, Gravity: An Introduction to Einstein's General Relativity, 2003.

Derivation of RDM equations, computer algebra

- complex calculations, for example, Riemann tensor 4^4= 256 components
- substitution, differentiation, algebraic simplifications
- convenient to use a system of analytical computations
- example calculation in Mathematica

Algorithm geodesic(n,x,u,gam):

```
rhoeq = Simplify[
          Sum[ u[[i]] D[rho[r],x[[i]]], {i,1,n} ]
        + rho[r] ( Sum[ D[u[[i]],x[[i]]], {i,1,n} ]
        + Sum[ gam[[i,i,k]] u[[k]],
          {i,1,n},{k,1,n} ] )
        ];
geoeq = Simplify[ Table[
          Sum[ u[[j]] D[u[[i]],x[[j]]], {j,1,n} ]
        + Sum[ gam[[i,j,k]] u[[j]] u[[k]],
          {j,1,n},{k,1,n} ],
        {i,1,n} ] ]
```

$$(u^\nu \partial_\nu) u^\mu + \Gamma^\mu{}_{\nu\lambda} u^\nu u^\lambda = 0,$$

$$u^\mu \partial_\mu \rho + \rho(\partial_\mu u^\mu + \Gamma^\mu{}_{\mu\lambda} u^\lambda) = 0$$

Derivation of RDM equations, computer algebra

The result of substitutions, geodesic eqs for RDM:

$$(\rho u^r)' + (4/r + A'/A + B'/B)\rho u^r/2 = 0,$$
$$(u^t A'/A + (u^t)')u^r = 0,$$
$$(u^t)^2 A' + (u^r)^2 B' + 2Bu^r (u^r)' = 0,$$

Analytical solution:

$$\rho = c_1 / \left(r^2 u^r \sqrt{AB} \right),$$
$$u^t = c_2/A,\ \ u^r = \sqrt{c_2^2 + c_3 A}/\sqrt{AB}$$

$c_{1,2,3}$ – integration consts,
$c_{1,2}>0$, $c_3=-1,0,+1$

Derivation of RDM equations, computer algebra

Einstein's equations for RDM model

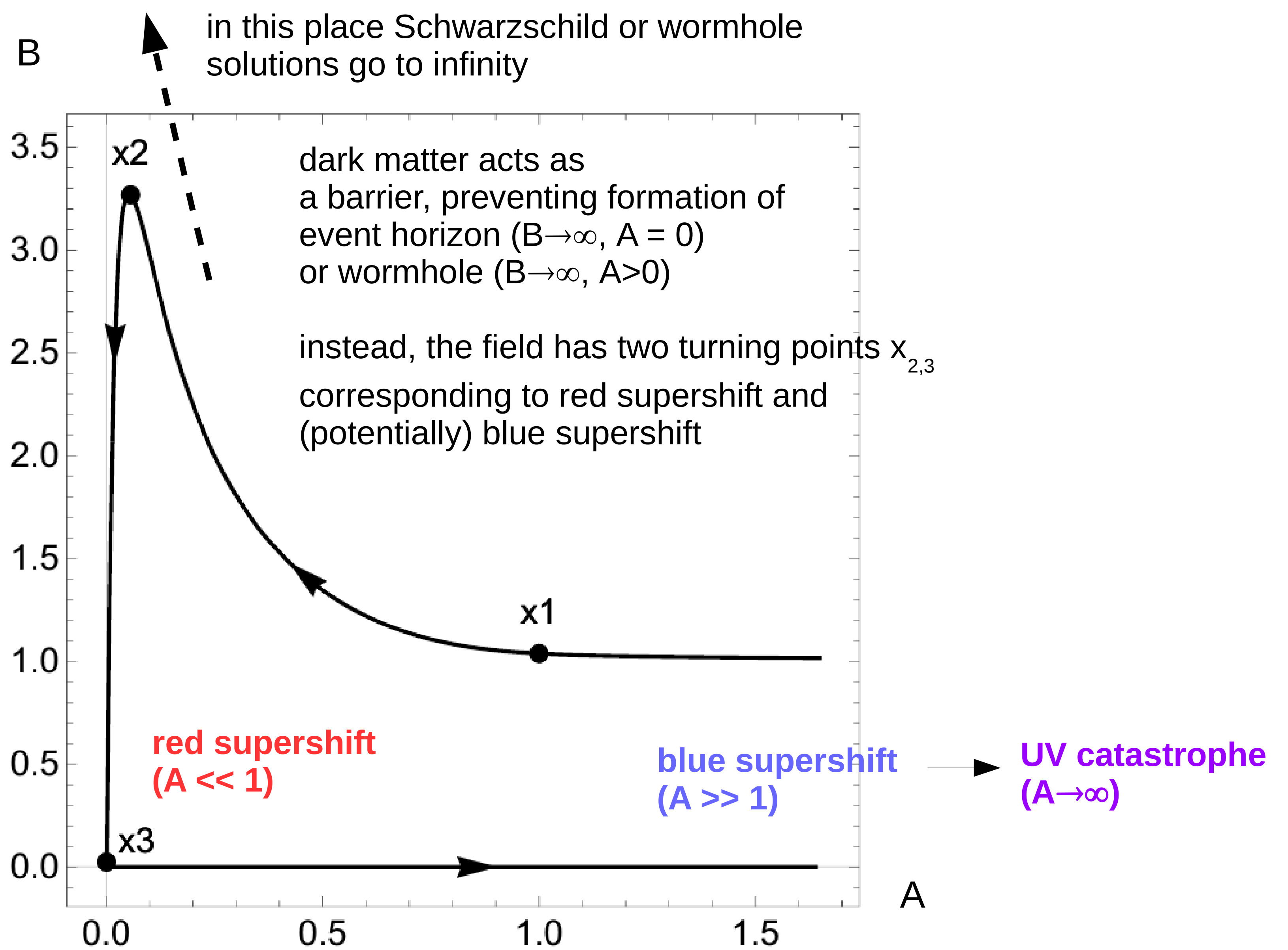
$$rA' = -A + AB + 4c_1 B\sqrt{c_2^2 + c_3 A},$$
$$rB' = B/A\left(A - AB + 4c_1 c_2^2 B/\sqrt{c_2^2 + c_3 A}\right)$$

in the limiting case $c_1 = 0$, dark matter switched off, the analytical solution in the form of a Schwarzschild black hole

in general, there is no analytical solution (system solved numerically)

Mathematica NDSolve

The numerical solution of RDM equations



The numerical solution of RDM equations

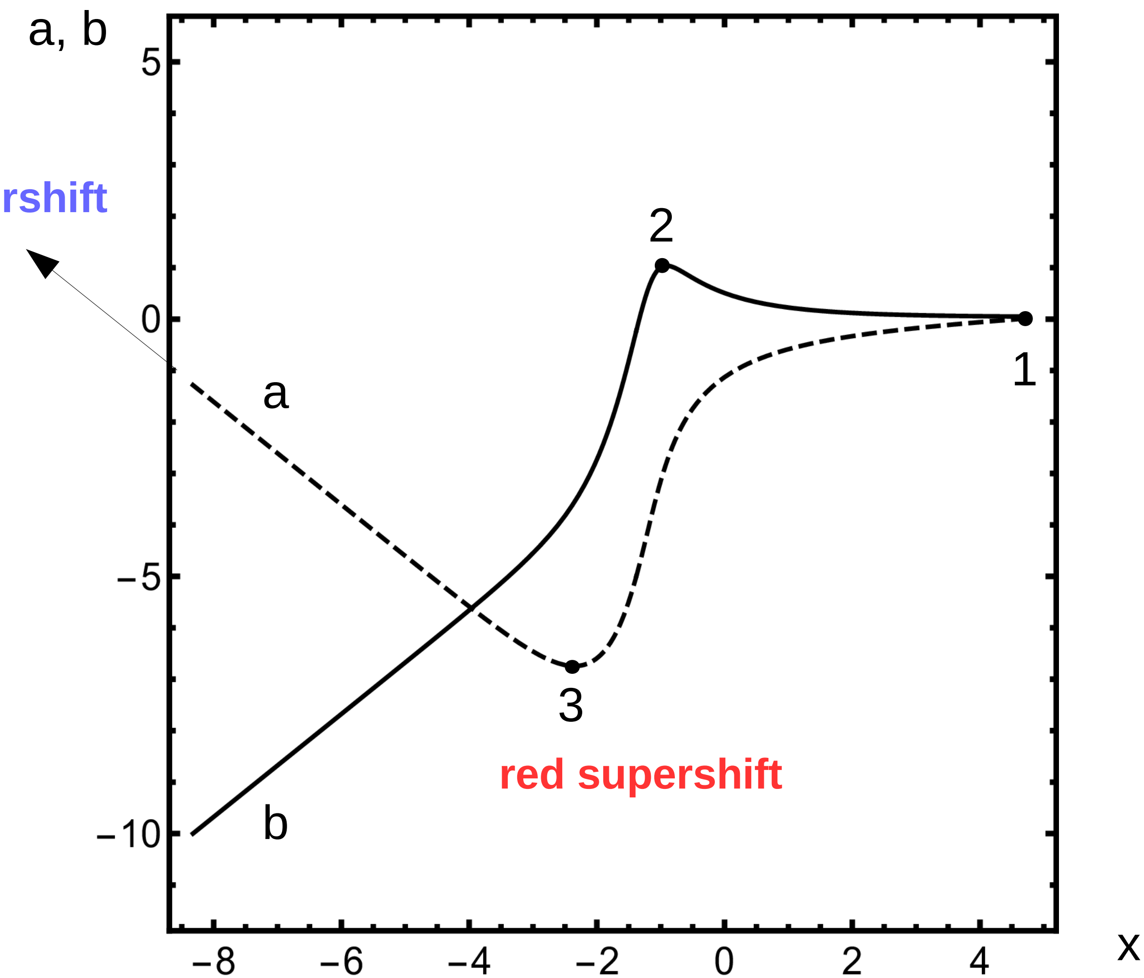
details in the log. scale

to improve the accuracy of integration the autonomy of the system is used and the system is reduced to a single eq db/da=f(a,b)

due to nonmonotonicity of solution, it is necessary to change the integration variable a <-> b at an intermediate point

relative accuracy of integration ~ 10^{-6}

time of solution 0.006sec (3GHz CPU)



$$x = \log r,\ \ a = \log A,\ \ b = \log B$$

The physical meaning of the constants

$$c_3 = u_\mu u^\mu = -1, 0, +1$$

matter type: massive, null, tachyonic (M/N/T-RDM)

solution *in strong fields (A<<1)* does not depend on matter type, since the term $c_3 A$ becomes small

solution *in weak fields (A~1)* depends on combination of constansts:

$$c_4 = 4c_1c_2,\ c_5 = c_3/c_2^2,$$

$$c_6 = c_4\sqrt{1+c_5},\ c_7 = c_4/\sqrt{1+c_5},$$

parameter c_5 defines *asymptotic radial velocity* of dark matter:
$c_5<-1$, MRDM flow has a turning point, the matter cannot escape
$c_5>-1$, all matter types, the matter can escape to large distances (the case further considered)

$$\epsilon = (c_6 + c_7)/2$$

parameter, defining *asymptotic gravitating density* ($\rho_{eff}+p_{eff}$) of dark matter flow

$$\rho_{\rm eff} = c_4/(2r^2)/\sqrt{1+c_5}, \quad p_{\rm eff} = c_4/(2r^2)\cdot\sqrt{1+c_5}$$

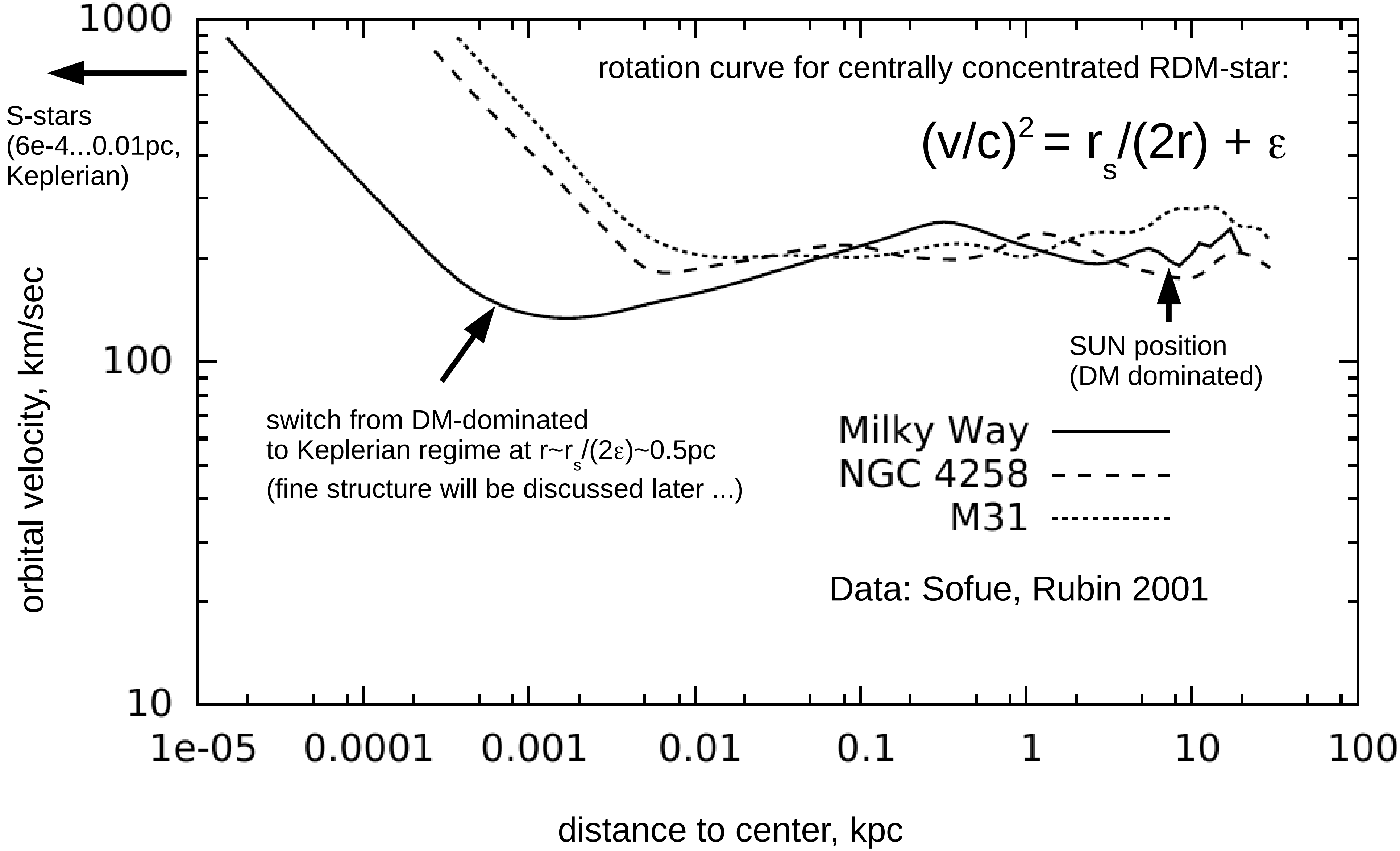
directly measurable parameter: $\varepsilon = (v/c)^2$, where v is the orbital velocity of stars at large distances from the galaxy center, for Milky Way v ~ 200 km/s, $\varepsilon = 4\times10^{-7}$

Comparison of RDM model with parameters of Milky Way

model parameters	$\epsilon = 4 \cdot 10^{-7}$, $r_0 = 1.2 \cdot 10^{10}$ m	
a border of the galaxy (starting point)	$r_1 = 3.1 \cdot 10^{21}$ m, $a_1 = 0$, $b_1 = 2.67 \cdot 10^{-7}$	100kpc DM domination
data at Earth location	$r_E = 2.57 \cdot 10^{20}$ m, $a_E = -2 \cdot 10^{-6}$, $b_E = b_1 + 4 \cdot 10^{-11}$	Sun-Earth 8.3kpc
switch from DM-dominated to Keplerian regime	$r_{1a} = 1.5 \cdot 10^{16}$ m, $a_{1a} = -1.05 \cdot 10^{-5}$, $b_{1a} = 9.91 \cdot 10^{-7}$	S-stars Kepl.orbits $10^{13\text{-}15}$m
switch to Schwarzschild regime	$r_{1b} = 3.33 \cdot 10^{10}$ m, $a_{1b} = -b_{1b} - 2.06 \cdot 10^{-5}$, $b_{1b} = 0.404$	circular orbits become instab.
begin of the supershift	$r_2 = 1.11 \cdot 10^{10}$ m, $a_2 = -14.79$, $b_2 = 13.40$	gravit. radius
switch of integration $b(a) \to a(b)$	$r_{2a} = r_2 - 1.2 \cdot 10^4$ m, $a_{2a} = -16.79$, $b_{2a} = 12.54$	
end of the supershift	$r_3 = 6.8 \cdot 10^6$ m, $a_3 = b_3 - 14.79$, $b_3 = -1.33 \cdot 10^6$	supershift (mass inflation)
redshift at the minimal radius (Planck length)	$r_{Pl} = 1.62 \cdot 10^{-35}$ m, $a_{Pl}/a_3 - 1 = -7.19 \cdot 10^{-5}$	naked singularity

=> supershift remains **red** until Planck length (no UV-catastrophe)

Comparison of RDM model with parameters of Milky Way



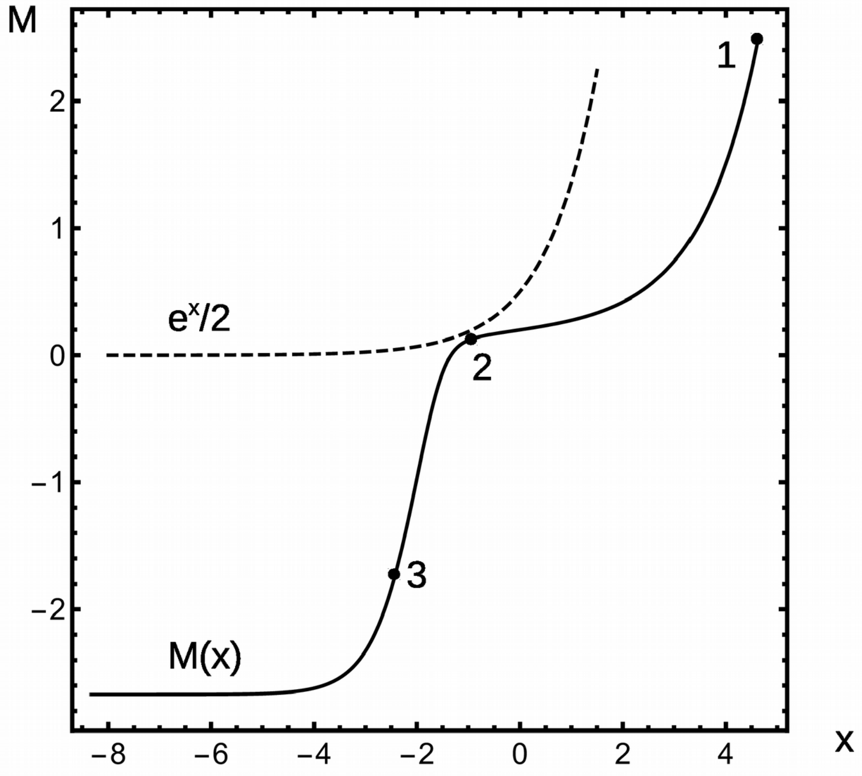
The RDM-stars as black and white holes

- RDM-stars have both properties of black and white holes, as they are permanently *absorb and emit* spherical shells of dark matter
- T-symmetric stationary solution analogous to Planck stars with permanently repeating QG-bounce
- also have **negative mass** in the center

$$M = r/2\ (1 - B^{-1})$$

Misner-Sharp mass
(1) decreases with decreasing r, when positive mass layers removed from the star ...
(2) when approaching the horizon (2M = r), decreases faster 2M <r, the horizon is erased ...
(3) decreases very rapidly in supershift region, **mass inflation** (Hamilton, Pollack 2005)

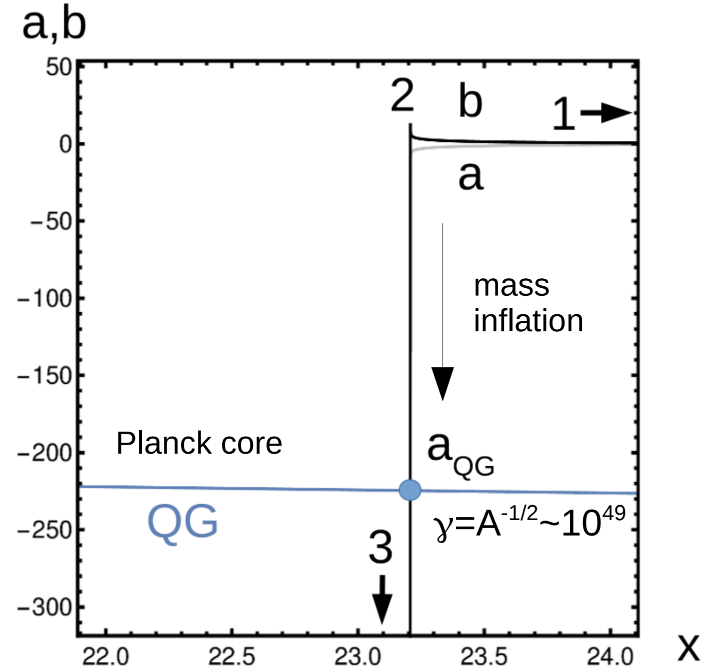
=> **central value M(0) <0**



Negative masses

- Energy conditions (Einstein, Hawking): there are *no* negative masses
- 't Hooft (1985): "... negative mass solutions unattractive to work with but *perhaps they cannot be completely excluded*."
- Visser (1996): negative masses *are needed* to create the wormholes and time machines
- Rovelli-Vidotto (2014), Barceló et al. (2015): negative masses *can be obtained* effectively by a slight excess of Planck density

- *Specifically, for RDM-stars:* relative excess of Planck density **$\Delta\rho / \rho_P \sim 3\varepsilon$** provides a hydrostatic equilibrium for galactic dark matter halo; $\varepsilon = 4x10^{-7}$ for MW

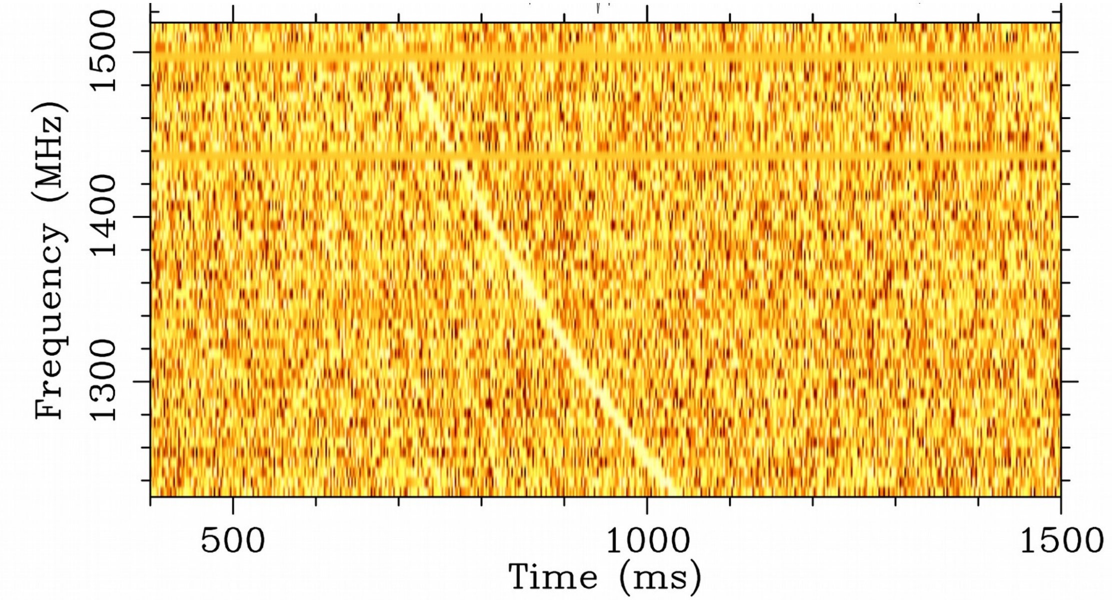


calculation for the Milky Way galaxy

Experiment: fast radio bursts

Fast Radio Bursts (FRB), powerful flashes of extragalactic origin

Big Scanning Antenna (BSA), Pushchino, Russia, registered 3 FRB of lowest frequency **111MHz**

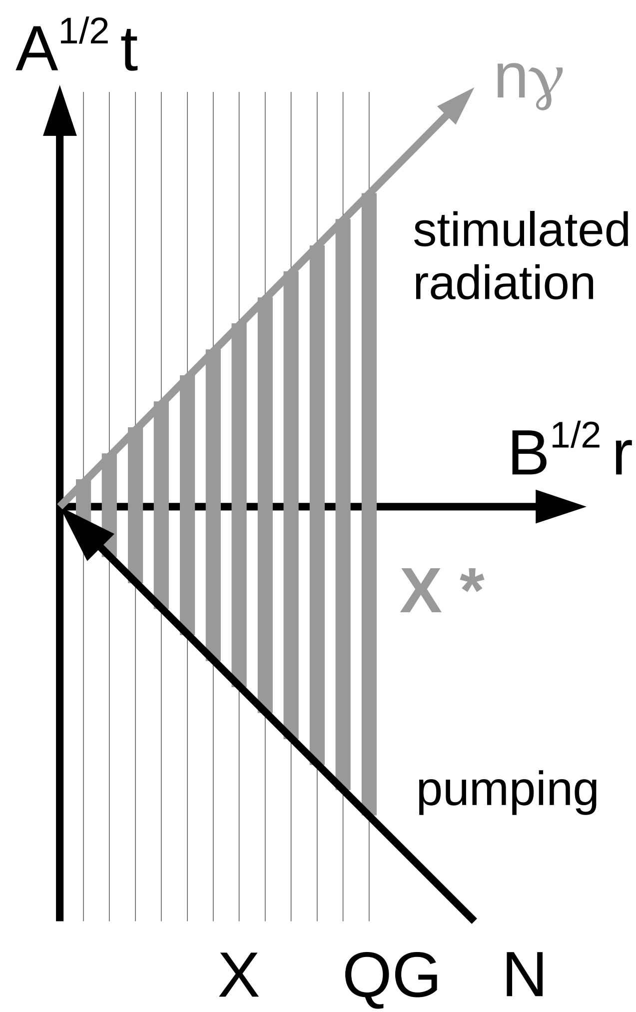


typical signature of FRB (the first registered flash FRB010724, Lorimer et al. 2007, frbcat.org) the slope indicates high dispersion shift (extragalactic distance)

- reported totally 84 FRB sources, 2 of which are repeating (data of 16.06.19)
- duration: *0.08msek (fast) -5sek,* frequency: **111MHz-8GHz** (radio band)
- typical isotropic energy of the flash ~ $10^{32\text{-}34}$J, corresp. $E = mc^2$ for a small asteroid
- the nature of bursts is currently unknown

Experiment: fast radio bursts

- FRB generation mechanism in RDM model
- object of an asteroid mass falls onto the RDM-star
- grav. field acts as an accelerator with super-strong **ultrarelativistic factor $\gamma \sim 10^{49}$**
- nucleons N composing the asteroid enter in the inelastic collisions with particles X forming the Planck core, producing the excited states of a typical energy **$E(X^*) \sim \mathrm{sqrt}(2m_X E_N)$**
- high-energy photons formed by the decay of X* $E(\gamma,in) \sim E(X^*)/2$ are subjected to super-strong **red shift factor γ^{-1}**
- outgoing energy $E(\gamma,out) \sim \mathrm{sqrt}(m_X m_N/(2\gamma))$, wavelength $\lambda out = \mathrm{sqrt}(2\lambda_X \lambda_N \gamma)$, where $\lambda_X \sim 1.6\times10^{-35}$m (Planck length), $\lambda_N \sim 1.32\text{x}10^{-15}$m (Compton wavelength of nucleon)
- $\lambda out = 2\,(2\pi)^{1/4}\,\mathrm{sqrt}(r_s \lambda_N)/\varepsilon^{1/4}$, for Milky Way parameters $r_s = 1.2\text{x}10^{10}$m, $\varepsilon = 4\text{x}10^{-7}$, $\lambda out = 0.5$m, $\nu out =$ **600MHz**
- falls in the observed range **111MHz-8GHz**



- a common mechanism of stimulated emission (aka LASER) generates a short pulse of *coherent* radiation

Experiment: rotation curves of galaxies

- universal rotation curve (URC, Salucci et al. 1995-2017)

- represents averaged exp. rotation curves of >1000 galaxies
- before averaging: galaxies are subdivided to bins over magnitude mag
- curves V(R,mag) are normalized to the values at optical radius: V / Vopt, R / Ropt
- the averaging smoothes the individual characteristics of the curves (loc. minima / maxima)

- detailed modeling of rotation curves in RDM model
- based on the assumptions: (1) all black holes are RDM-stars; (2) their density is proportional to the concentration of the luminous matter in the galaxy
- in this case, the dark matter density is given by the integral (Kirillov, Turaev 2006)

constant

$$\rho_{dm}(x) = \int d^3x'\, b(|x-x'|)\, \rho_{lm}(x'), \quad b(r) = 1/(4\pi L_{KT})/r^2$$

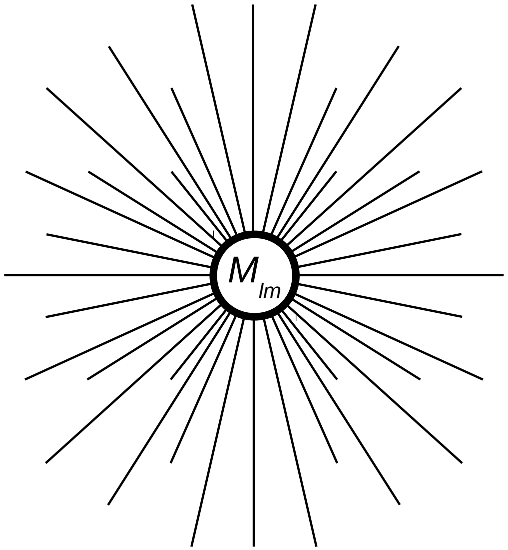
Freeman 1970 model → ρ_{lm}

the contribution of one RDM-star

$$\sim \delta(z) \exp(-r/R_D),\, R_{opt} = 3.2 R_D$$

← optical radius of the galaxy encompassing 83% of the light

Experiment: rotation curves of galaxies



The physical meaning of KT-integral: every element of luminous matter (i.e., RDM-stars contained in it) gives additive contributions to dark matter density, mass, gravitational field, orbital velocity, gravitational potential...

$$\rho_{dm}(r) = M_{lm}/(4\pi L_{KT})/r^2,$$

$$M_{dm}(r) = 4\pi \int_0^r dr' r'^2 \rho_{dm}(r') = M_{lm} r / L_{KT},$$

$$a_{r,dm} = G M_{dm}(r)/r^2 = G M_{lm}/(r L_{KT}),$$

$$v_{dm}^2 = G M_{dm}(r)/r = G M_{lm}/L_{KT},$$

$$\varphi_{dm} = G M_{lm}/L_{KT} \cdot \log r.$$

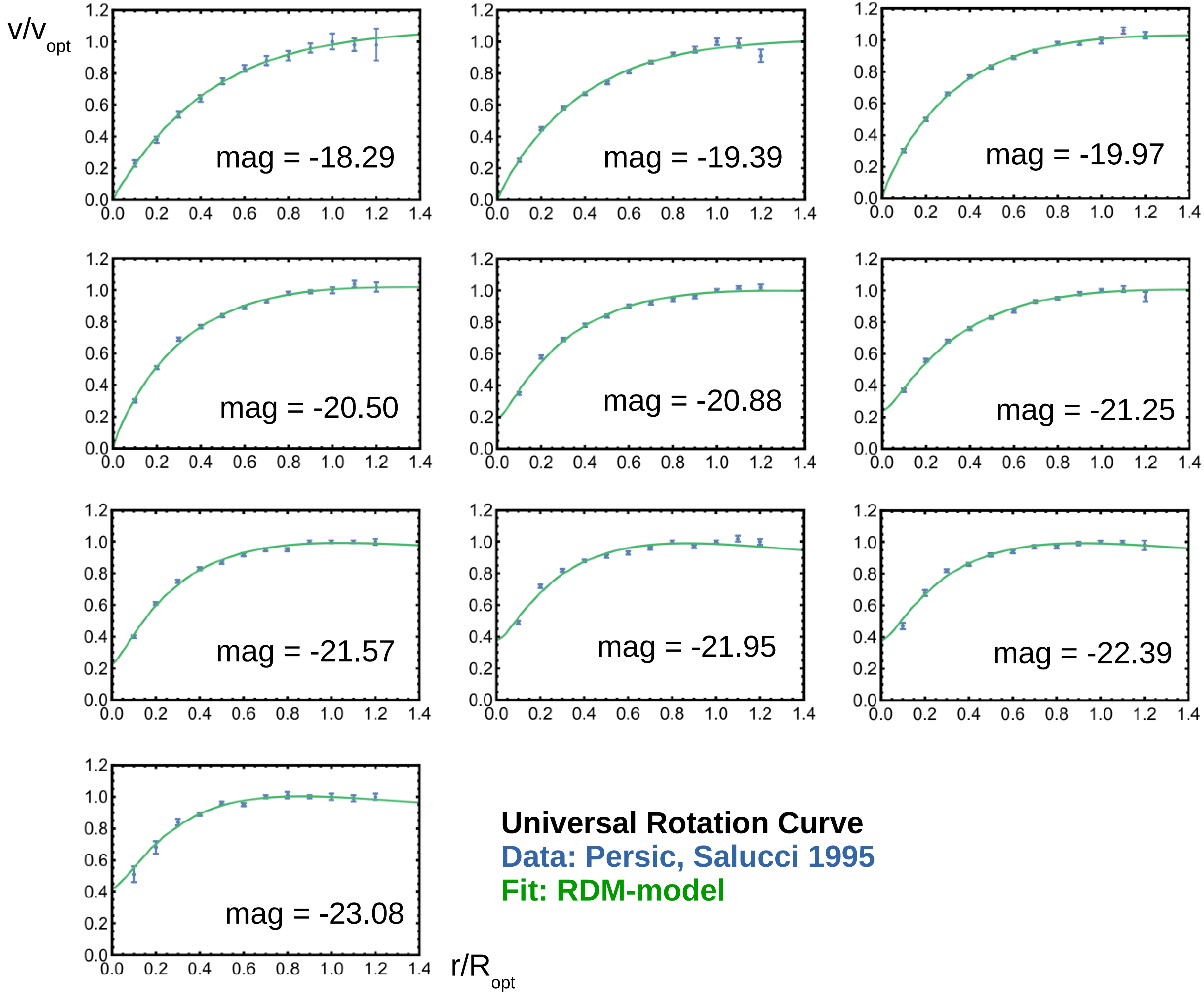
LKT is the distance at which the mass of dark matter equals to the mass of the luminous matter, to which it is coupled

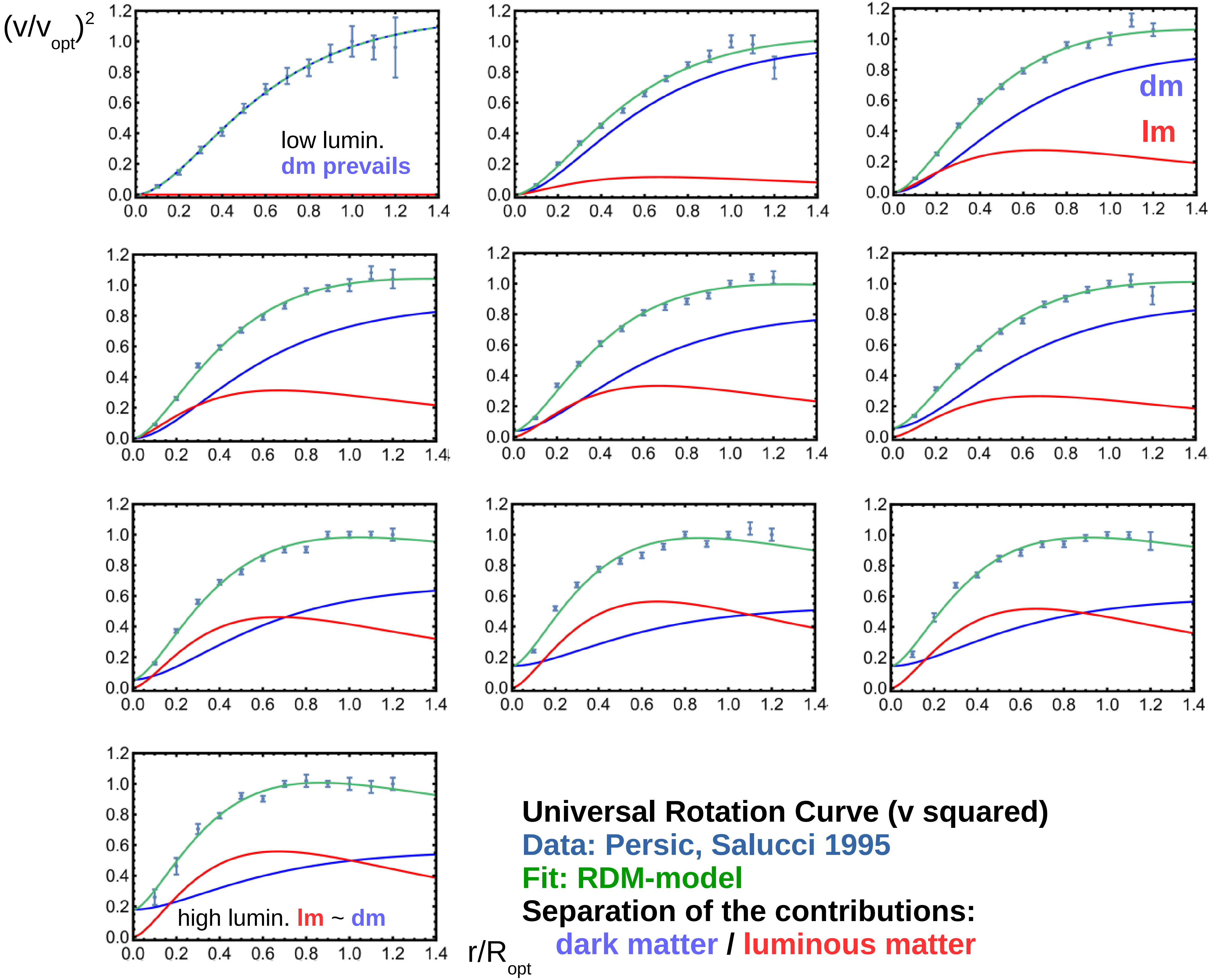
Experiment: rotation curves of galaxies

the integrals are evaluated analytically and lead to the following model:

$$v^2_{center,lm} = \alpha_0 v^2_{opt} R_{opt}/r, \;\; v^2_{center,dm} = \alpha v^2_{opt},$$
$$v^2_{disk,lm} = \beta v^2_{opt} F_{disk}(r/R_D)/F_{disk}(3.2),$$
$$F_{disk}(x) = x^2(I_0(x/2)K_0(x/2) - I_1(x/2)K_1(x/2)),$$
$$v^2_{disk,dm} = \gamma v^2_{opt} F_{disk,dm}(r/R_D)/F_{disk,dm}(3.2),$$
$$F_{disk,dm}(x) = 1 - e^{-x}(1+x), \;\; R_{opt} = 3.2 R_D,$$
$$v^2 = v^2_{center,lm} + v^2_{center,dm} + v^2_{disk,lm} + v^2_{disk,dm},$$
$$v^2_{opt} = v^2(r \to R_{opt}), \;\; \alpha_0 + \alpha + \beta + \gamma = 1,$$

the contributions are separated for the galactic center (unresolved), disk, visible and dark matter; I_n, K_n - modified Bessel functions; coeff. at basis shapes selected as fitting parameters


v/v
opt
mag = -18.29
mag = -19.39
mag = -19.97
mag = -20.50
mag = -20.88
mag = -21.25
mag = -21.57
mag = -21.95
mag = -22.39
mag = -23.08
r/R
opt
Universal Rotation Curve
Data: Persic, Salucci 1995
Fit: RDM-model



Universal Rotation Curve (v squared)

Data: Persic, Salucci 1995

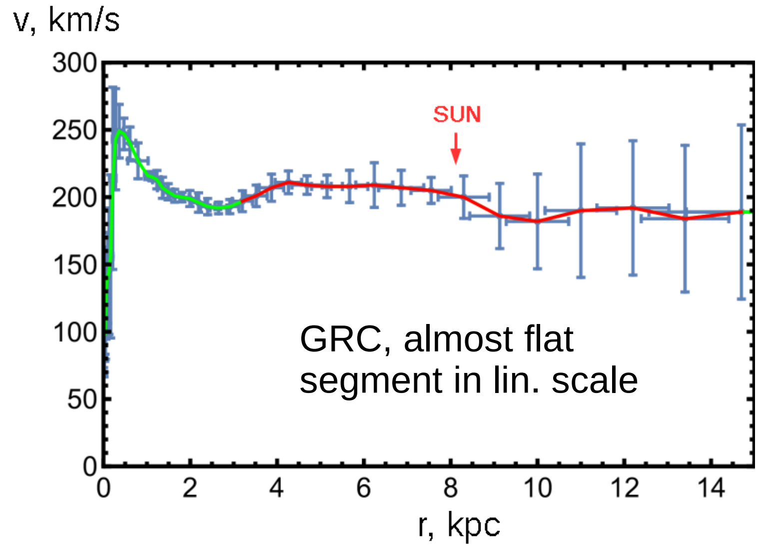
Fit: RDM-model

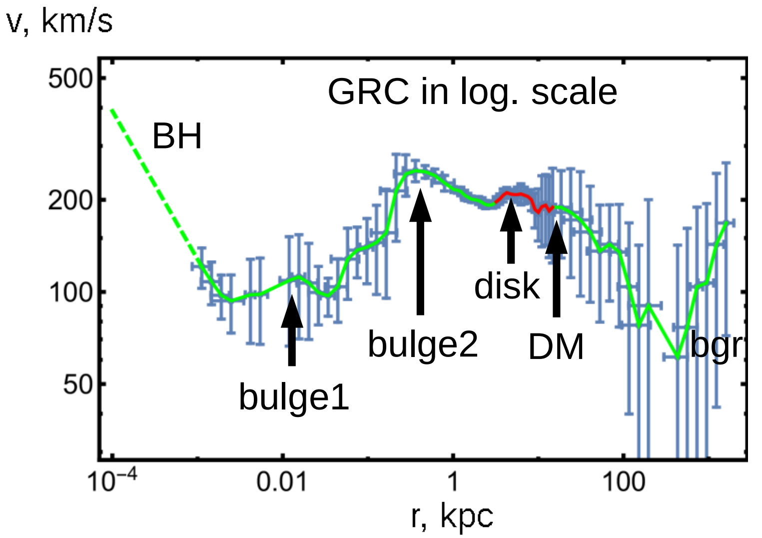
Separation of the contributions:

dark matter / luminous matter

Experiment: rotation curves of galaxies

- rotation curve for the Milky Way in a large range of distances (Grand Rotation Curve, GRC, Sofue et al. 2009-2013)
- shows individual structures typical for a particular galaxy (MW)
- structures are clearly visible in log.scale (central black hole, the inner, outer bulges, disk, dark matter halo, the background contribution)
- in the fitting procedure each structure is represented by its own basis function
- we consider several scenarios with fixed coupling constants of dark matter to separate structures





λ_{KT}	s1	s2	s3
λ_{bh}	0	1	10^3
λ_1	0	1	10^2
λ_2	0	1	2
λ_{disk}	1	1	1

$$v_{bh}^2 = G_m M_{bh}/r,\ \ G_m = 4.3016 \cdot 10^{-6}\ (km/s)^2 (kpc/M_\odot),$$
$$v_{sph,i}^2 = G_m M_i/r \cdot F_{sph}(r/a_i),\ \ i = 1, 2,$$
$$F_{sph}(x) = 1 - e^{-x}(1 + x + x^2/2),$$
$$v_{disk}^2 = G_m M_{disk}/(2R_D) \cdot F_{disk}(r/R_D),$$
$$F_{disk}(x) = x^2 (I_0(x/2)K_0(x/2) - I_1(x/2)K_1(x/2)),$$
$$v_{lm}^2 = v_{bh}^2 + v_{sph1}^2 + v_{sph2}^2 + v_{disk}^2,$$
$$v_{dm,bh}^2 = G_m M_{bh} \lambda_{bh}/L_{KT},$$
$$v_{dm,sph,i}^2 = G_m M_i \lambda_{sph,i}/L_{KT} \cdot F_{dm,sph}(r/a_i),\ \ i = 1, 2,$$
$$F_{dm,sph}(x) = (6x + (3 - 3x + x^2)e^x Ei(-x)$$
$$-(3 + 3x + x^2)e^{-x} Ei(x))/(4x),$$
$$v_{dm,disk}^2 = G_m M_{disk} \lambda_{disk}/L_{KT} \cdot F_{dm,disk}(r/R_D),$$
$$F_{dm,disk}(x) = 1 - e^{-x}(1 + x),$$
$$v_{dm,sum}^2 = v_{dm,bh}^2 + v_{dm,sph1}^2 + v_{dm,sph2}^2 + v_{dm,disk}^2,$$
$$v_{dm,cut}^2 = v_{dm,sum}^2(r \to r_{cut}) \cdot r_{cut}/r,\ \ v_{bgr}^2 = 4\pi G_m \rho_0 r^2/3,$$
$$v_{dm}^2 = ((v_{dm,sum}^2)^p + (v_{dm,cut}^2)^p)^{1/p},$$
$$v^2 = v_{lm}^2 + v_{dm}^2 + v_{bgr}^2.$$

the contributions: central BH, 2-component bulge, disc, visible and dark matter, homogeneous background density

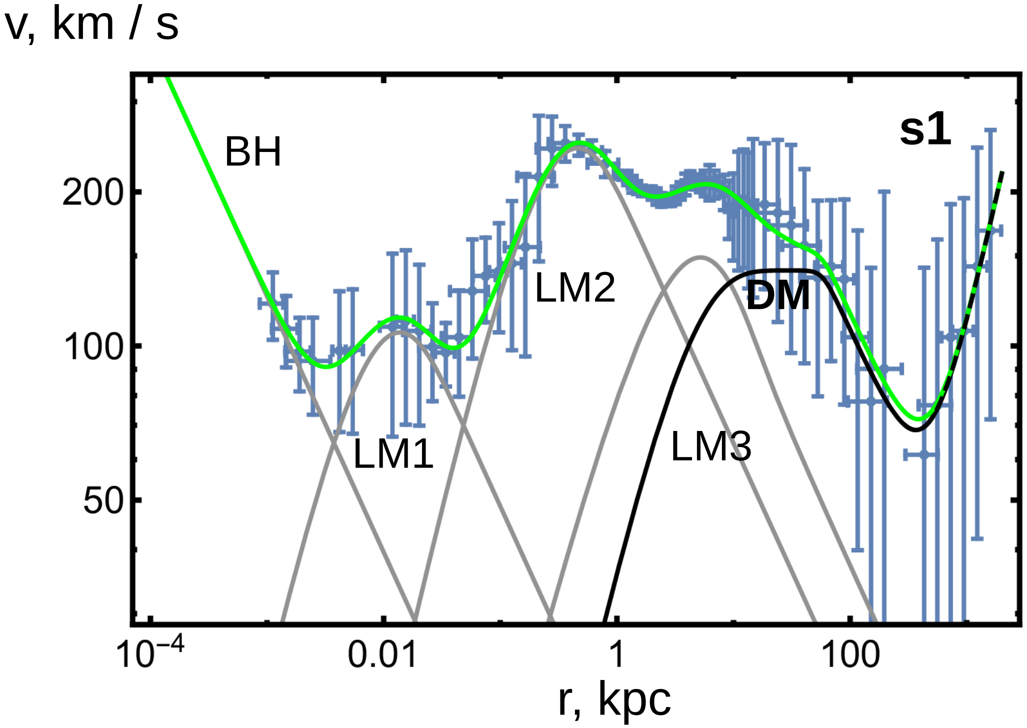
the fitting parameters: coefficients at basis shapes (or masses of components), geom. dimensions of the components

Ei - exponential integral function

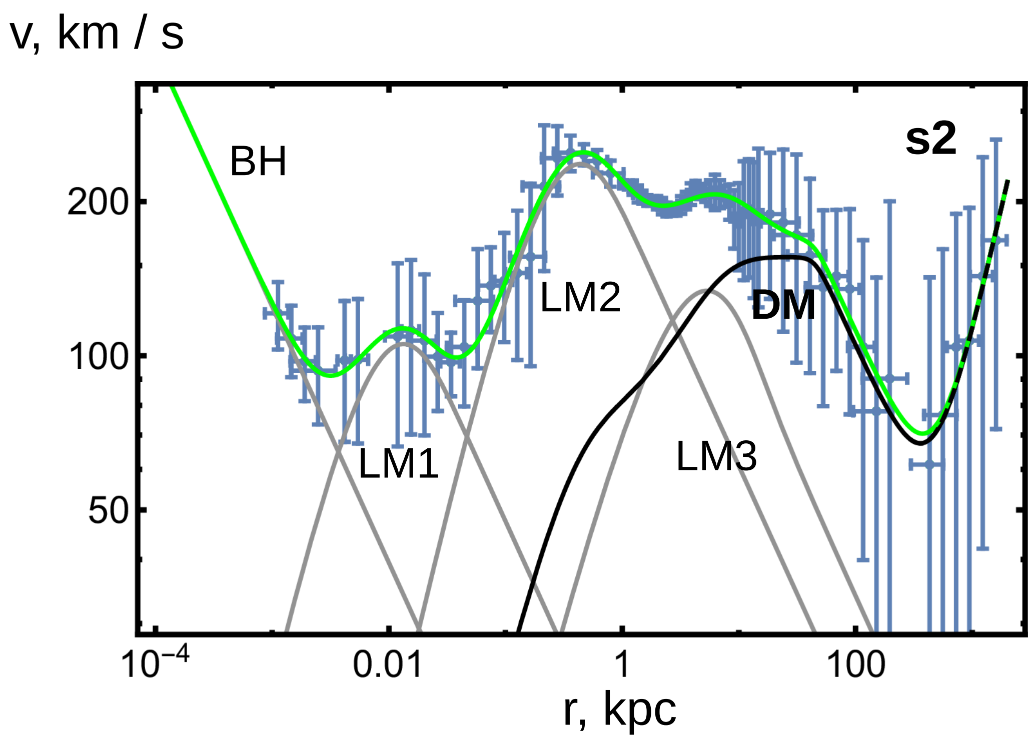
dark matter halo is cut on the outer radius rcut, analogous to termination shock phenomenon at the boundary of Solar sys., where solar wind stops meeting the uniform interstellar env.

background contribution, similar to uniform cosmological distr. (Hubble flow), with a possible local overdensity above the critical

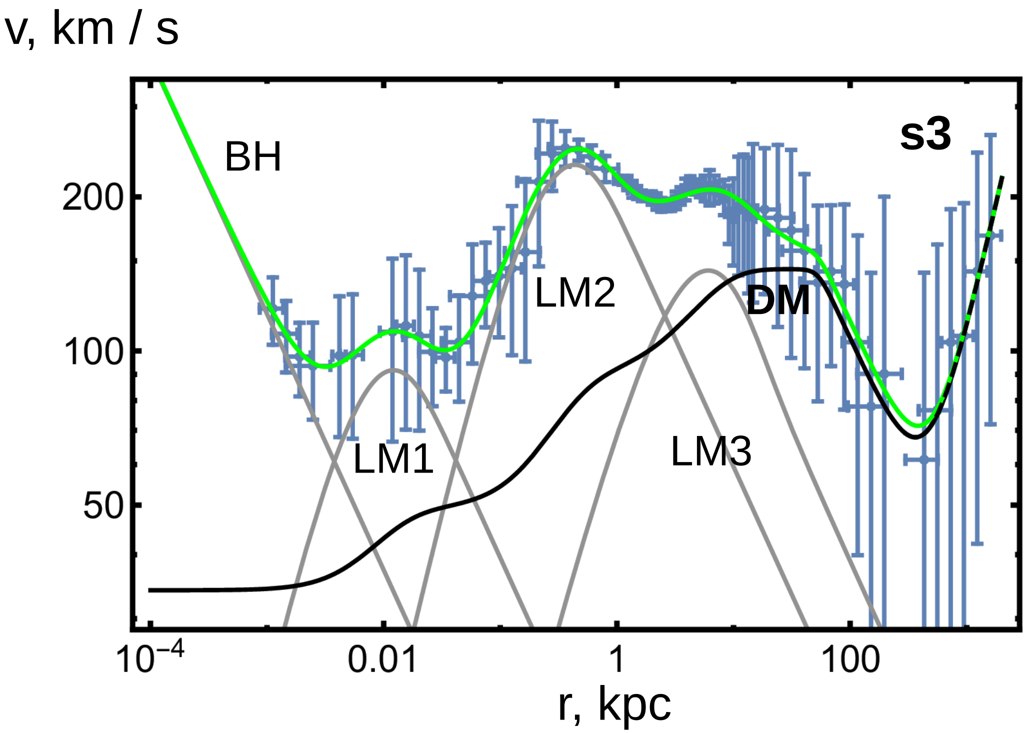
Experiment: rotation curves of galaxies



Experiment: rotation curves of galaxies



Experiment: rotation curves of galaxies



Experiment: rotation curves of galaxies

GRC: fitting results, central values of parameters*

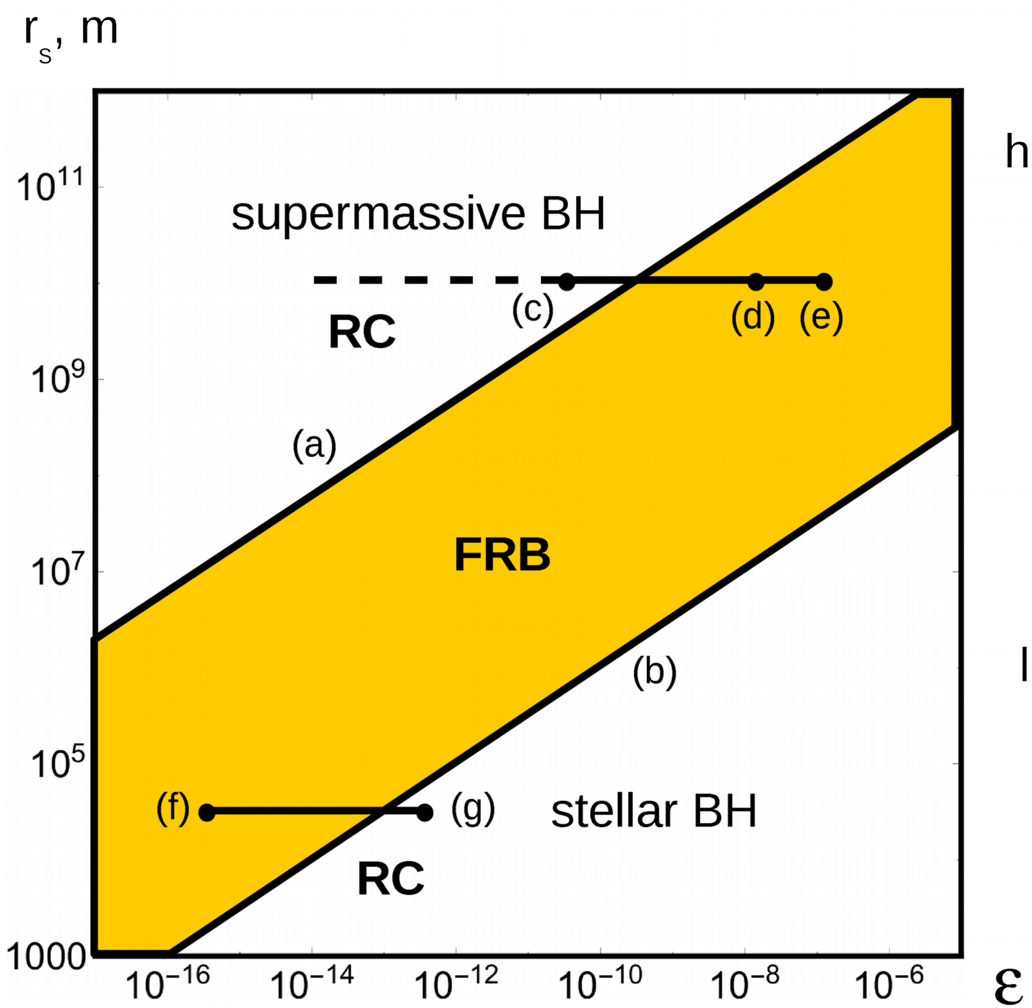
par	s1	s2	s3
M_{bh}	3.6×10^6	3.6×10^6	3.2×10^6
M_1	5.5×10^7	5.2×10^7	3.6×10^7
a_1	0.0041	0.0039	0.0036
M_2	9.7×10^9	8.6×10^9	8.2×10^9
a_2	0.13	0.13	0.13
M_{disk}	3.2×10^{10}	2.7×10^{10}	3.5×10^{10}
R_D	2.4	2.5	2.8
L_{KT}	7.0	6.3	12.0
r_{cut}	58	45	53
$M_{dm}(r_{cut})$	2.7×10^{11}	2.5×10^{11}	2.6×10^{11}
ρ_0	646	653	649

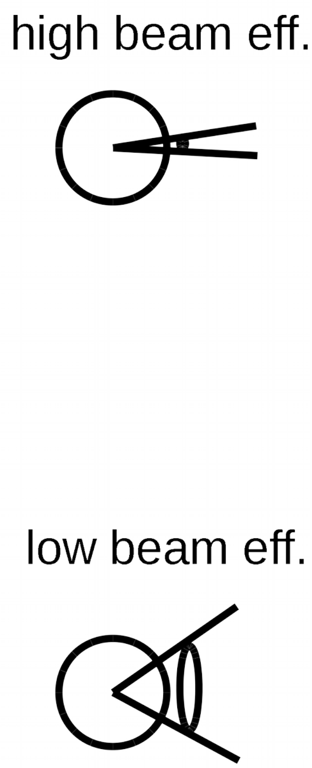
* masses in $M_\odot$, lengths in kpc, density in $M_\odot/kpc^3$

approx equal
for all scenarios

5x greater than
critical density
(local overdensity)

Combined analysis of FRBs and RCs

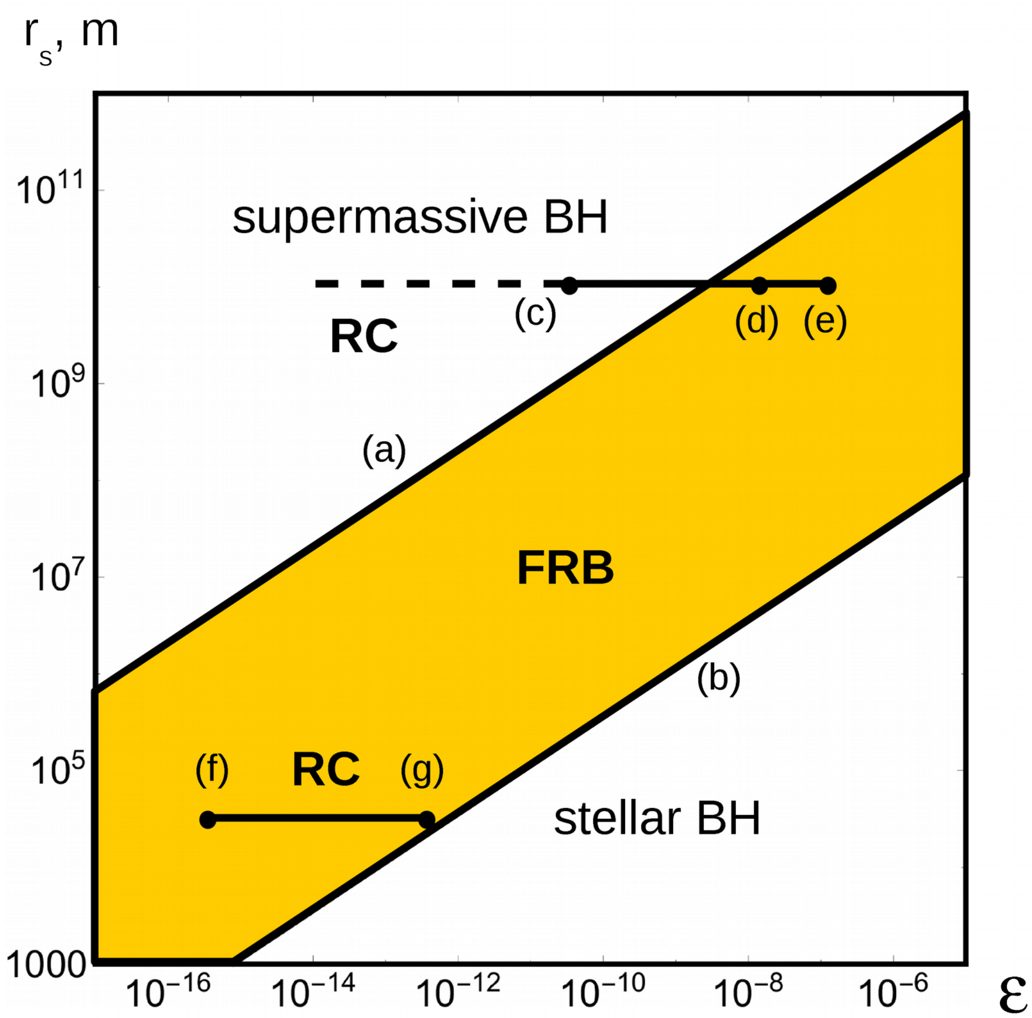




- two solutions for FRB sources: supermassive and stellar BH
- supermassive BH is preferable: high beam efficiency, high scatter broadening
- (Luan, Goldreich, 2014; Masui et al. 2015) (arXiv:1401.1795, arXiv:1512.00529) also attribute FRB source location to galactic nuclei

(a) νout=111MHz, (b) νout=8GHz, (c) MWs2,
(d) MWs3, (e) MWmax for V(smbh,dm)=100km/sec,
(f) $\varepsilon=4\times10^{-7}$ div to Nsbh=10^9, (g) same with Nsbh=10^6
(Wheeler, Johnson, 2011)(arxiv:1107.3165)

Combined analysis of FRBs and RCs



FRB adjustment factors:

earlier onset of QG effects: $\rho \rightarrow \rho_P/s_1$

nucleon fragmentation factor: $\lambda_N \rightarrow \lambda_N/s_2$

s_1=1, s_2=1/3 (constituent quarks)

(a) νout=111MHz, (b) νout=8GHz, (c) MWs2,
(d) MWs3, (e) MWmax for V(smbh,dm)=100km/sec,
(f) $\varepsilon=4\times10^{-7}$ div to Nsbh=10^9, (g) same with Nsbh=10^6
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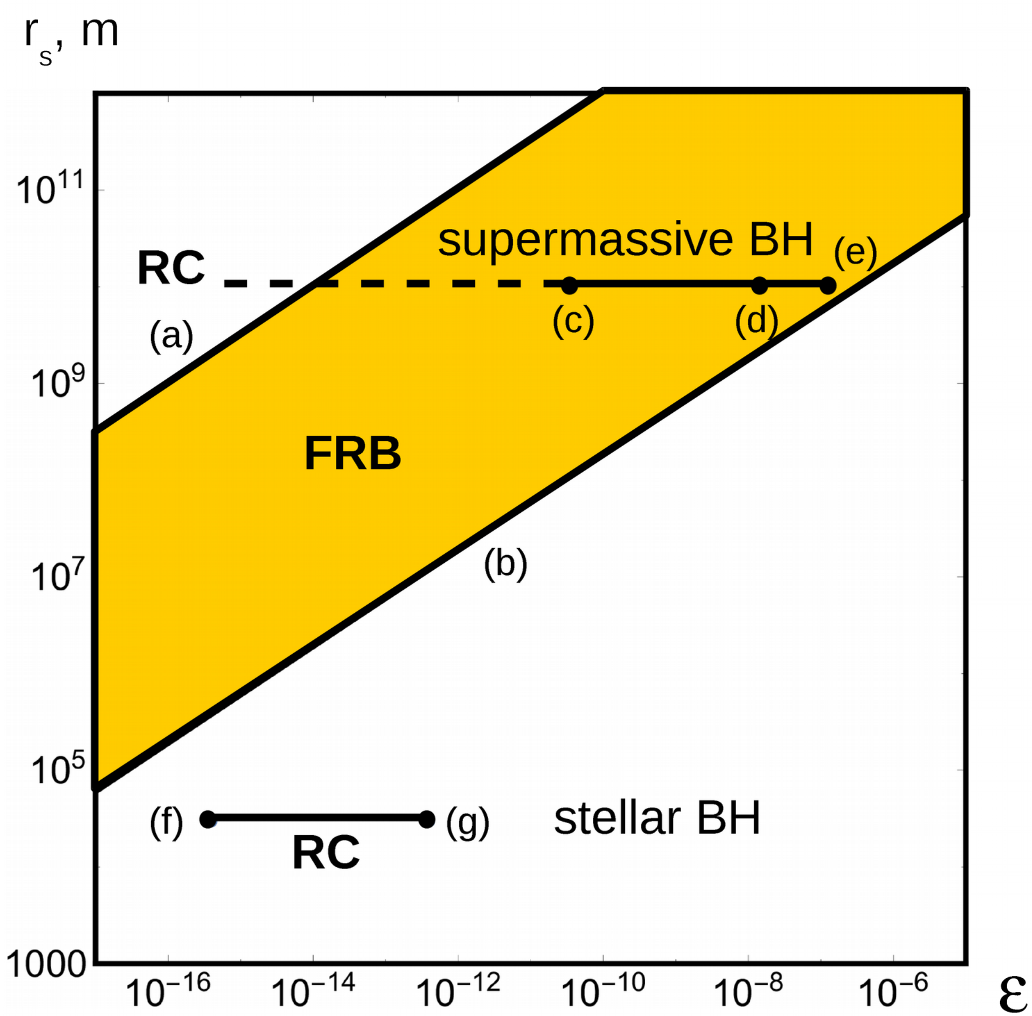
Combined analysis of FRBs and RCs



FRB adjustment factors:

earlier onset of QG effects: $\rho \rightarrow \rho_P/s_1$

nucleon fragmentation factor: $\lambda_N \rightarrow \lambda_N/s_2$

s_1=10, s_2=56 (iron nuclei)

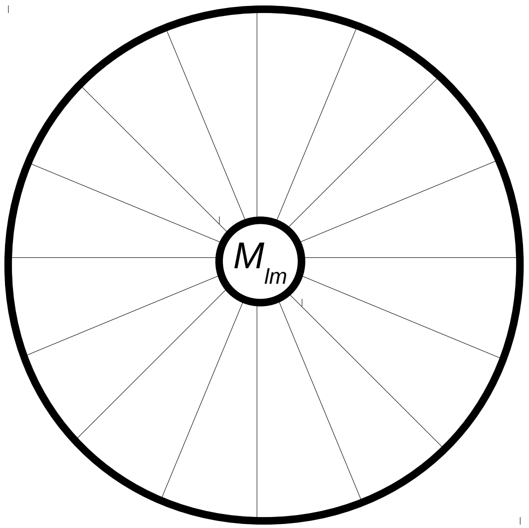
=> RDM descriptions of FRBs and RCs are compatible

(a) νout=111MHz, (b) νout=8GHz, (c) MWs2,
(d) MWs3, (e) MWmax for V(smbh,dm)=100km/sec,
(f) $\varepsilon=4\times10^{-7}$ div to Nsbh=10^9, (g) same with Nsbh=10^6
(Wheeler, Johnson, 2011)(arxiv:1107.3165)

Questions

Q1: Can *Tully-Fisher relation* be explained in RDM model?
Mlm ~ Vmax$^{\beta}$, β=4.48 $\pm$ 0.38 for stellar mass, β=3.64 $\pm$ 0.28 for total baryonic mass (Torres-Flores et al., 2011) (arXiv 1106.0505)

Hyp: a galaxy is formed by a collapse of matter in Rcut-sphere,
LM -> to the central region, DM -> to RDM configuration
=> Mlm/Mdm(Rcut)=Lκτ/Rcut=Ωlm/Ωdm=x~0.19



Check: x={0.12,0.14,0.23} for scenarios 1,2,3 **(ok)**

$Vmax^2$=G(Mlm+Mdm(Rcut))/Rcut=G(1+x)Mlm/Rcut

Mlm~Rcut^{D}, D=3, classical uniform distribution
D~2, *fractal distribution* (Mandelbrot 1997;
Labini, Montuori, Pietronero 1997; Kirillov, Turaev 2006)

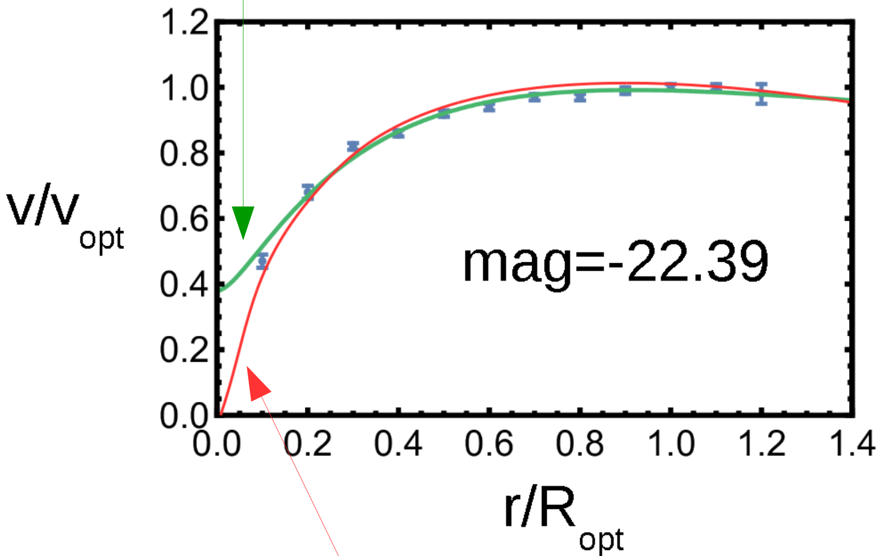
$Vmax^2$~Rcut^{D-1}~Mlm$^{(D-1)/D}$, **Mlm~Vmax**$^{2D/(D-1)}$
β=2D/(D-1), β=3 for D=3, β=4 for D=2

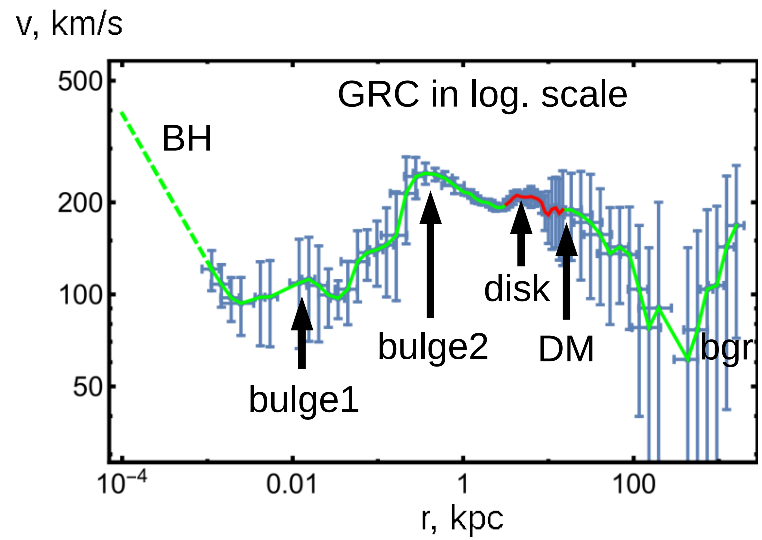
Q2: is it a coincidence that Lκτ/Ropt={0.9,0.8,1.3} ~ 1, i.e., Mdm(Ropt)~Mlm, for MW?

Questions

„cuspy“, RDM model with unresolved GC

here GC resolved:



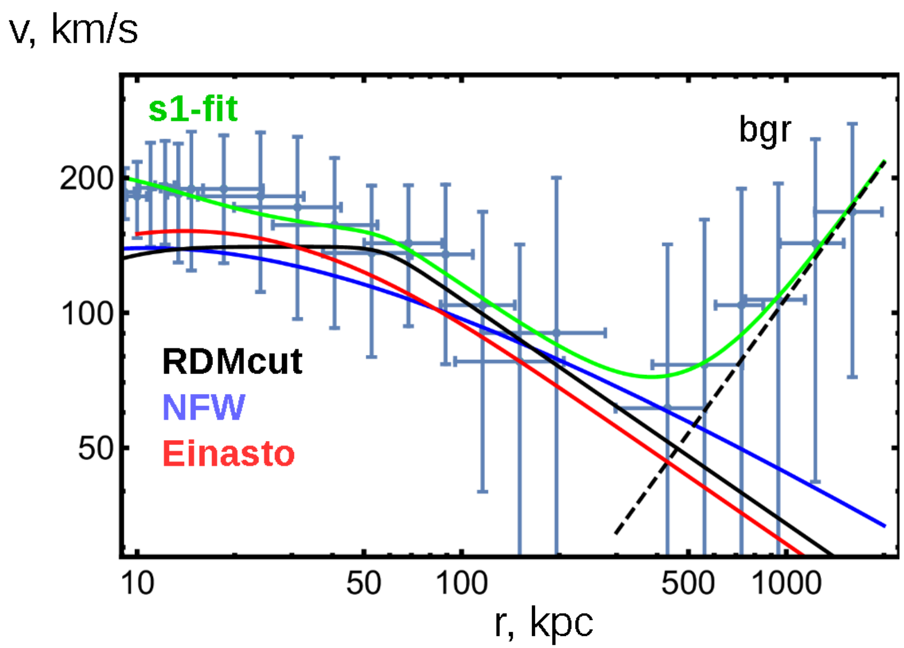


„cored“ (Persic, Salucci, Stel 1996)

GRC exp data show no tendency v->0 till central BH, non-cored?

URC differences in zero bin

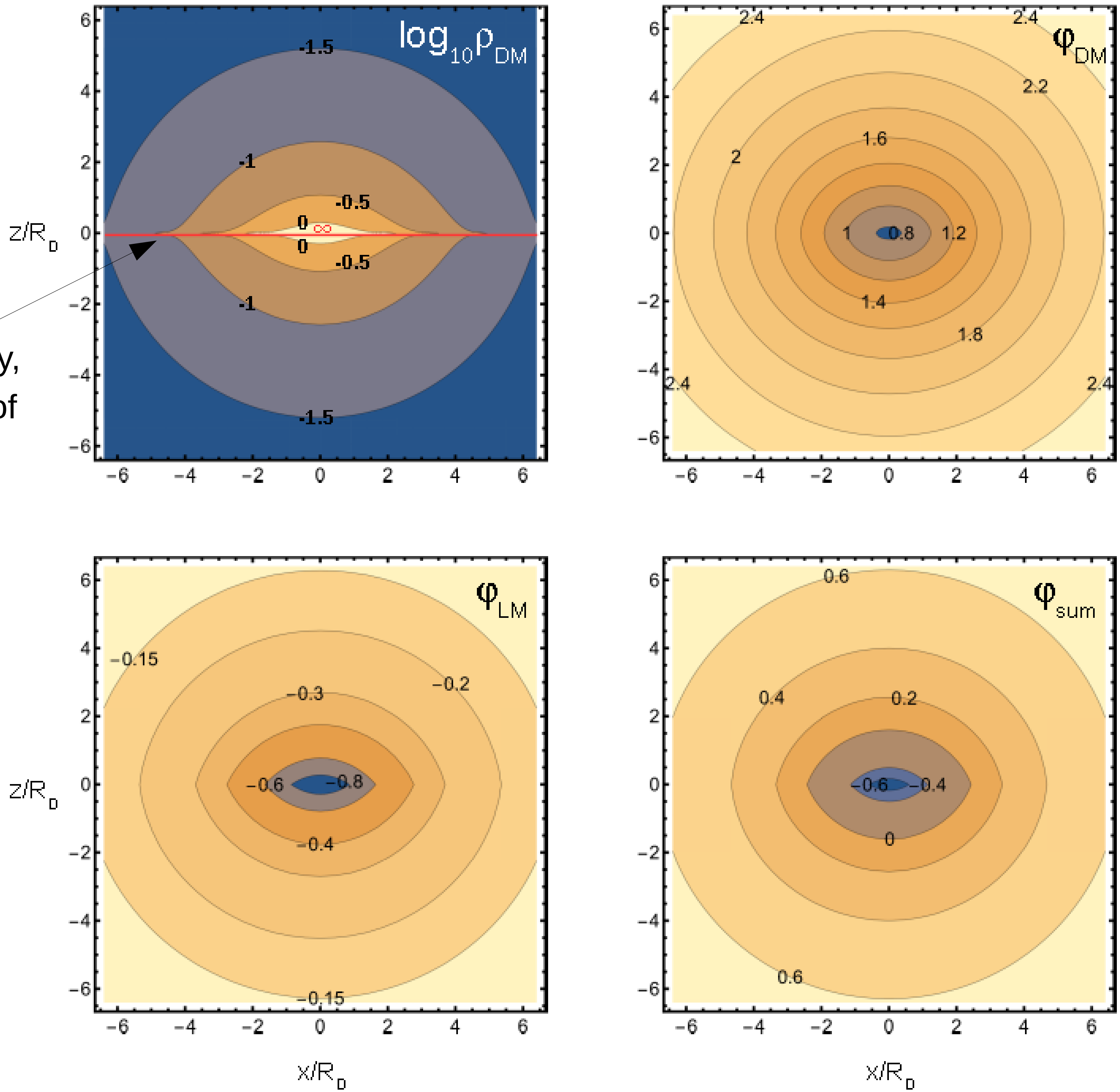
Questions



GRC outer part fitted by different profiles equally good
(can the scatter of data be reduced?)

hot radial dark matter produces the same exp observable rotation curves as **cold isotropic** dark matter, the difference is in switching on/off transversal pressure components, influencing solutions of field equations (proof in arXiv:1811.03368)

The distributions out of the galactic plane



integrable ρ_{DM}-singularity, the memory of $\rho_{LM} \sim \delta(z)$ in KT-integral (idealization)

aspherical DM-halo, becomes spherical at large distances (consequences for lensing experiments?)

$L_{KT}/R_D = 3$

Conclusion

- we have considered a number of astrophysical models: dark stars, wormholes, white holes, Planck stars ...
- considered in more detail the model of RDM-stars
- compared the model with experiment:

  fast radio bursts (FRB): the model correctly predicts the range of frequencies, the energies, the coherence properties of the signal

  galactic rotation curves are fitted well by the model prediction:
  the universal rotation curve describing the spiral galaxies of the general form and the individual rotation curve, describing the Milky Way galaxy in a wide range of distances.